* M B Z - 0 0 3 - 1 1 6 2 0 0 1 - 1 7 0 - 4 *

MBZ-003-1162001

Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

CMT - 2006 : Mathematics

(Algebra -II) (New Course)

Faculty Code : 003
Subject Code : 1162001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Each question carries 14 marks.

1 Answer following short questions : **7×2=14**

(i) For a ring R, define R–sub module of an R-module M.

(ii) Let M be an R-module. In std. notation, prove that $(-a)m = a(-m), \forall\ a \in R \text{ and } \forall\ m \in M$.

(iii) Let $f(x) = x^3 + 6x^2 + 7x + 8$. Prove that $f(x-2)$ is an irreducible polynomail. Is $f(x)$ irreducible? (Y/N)

(iv) State Eisenstein Criterion.

(v) For the field extension $^{R}|_{Q}$, write down two elements of R–Q, which are algebraic over Q and write down four elements of R, which are not algebraic over Q (they are transcendental elements over Q).

(vi) Write down the minimal polynomial of –i over Q.

(vii) Define splitting field.

2 Attempt any **two** : **2×7=14**

(a) Let $^{E}|_{F}$ *and* $^{K}|_{E}$ both are algebraic extensions. Prove that $^{K}|_{F}$ is also an algebraic extension.

(b) Prove that $Q\left(\sqrt{2}, \sqrt{3}, \ldots\ldots, \sqrt{p}, \ldots\right) | Q$ is an infinite algebraic extension.

(c) Let $p(x) \in F[x]$ be an irreducible polynomial and degree of $p(x) = n$. Let $^{E}|_{F}$ be an extension such that $\alpha \in E$ and α is a root of $p(x)$. Prove that $F[\alpha] = F(\alpha), [F(\alpha) : F] = n$ and $\{1, \alpha, \alpha^2, \ldots\ldots, \alpha^{n-1}\}$ is a basis of $F(\alpha)$ over F.

(d) Prove that every finite extension is an algebraic extension.

3 Attempt any **one** : **1×14=14**

(a) State and prove primitive element theorem.

(b) Let $^{E}|_{F}$ *and* $^{K}|_{E}$ both are finite separable extensions. Prove that $^{K}|_{F}$ is also a finite separable extension.

(c) Define algebraically closed field. For a field K, prove that following statements are equivalent :

(1) K is an algebraically closed field.

(2) If $p(x) \in K(x)$ and $p(x)$ is an irreducible polynomial, then degree of $p(x)$ is 1.

(3) Any $f(x) \in K[x]$, with degree of $f(x) \geq 1$, $f(x)$ can be split into linear factors in $K(x)$.

(4) Any $f(x) \in K[x]$, with degree of $f(x) \geq 1$, K contains all the roots of $f(x)$.

4 Attempt any **two** : **2×7=14**

(a) Let $f(x) \in F[x]$ be an irreducible polynomial. Prove that α is a multiple root of $f(x)$ if and only if $f'(x) = 0$ (All the coefficients of $f'(x)$ are multiple of char F.)

(b) Let char $k = p > 0$ and $f(x) \in k[x]$ be an irreducible polynomial. Prove that $f(x)$ has a multiple root if and only if $f(x) = g\left(x^{\rho}\right)$, for some $g(x) \in k[x]$.

(c) Let $f : M -\!\!\rightarrow N$ be an R–homomorphism of R–modules. Prove that Ker f and $f(M)$ are R–sub modules of M and N respectively.

(d) Prove that $G\left({}^{C}|_{R}\right)$ is a group of order 2.

(e) Let F be a finite field. Prove that $F^{*} = F - \{0\}$ is a cyclic group under multiplication.

5 Attempt any **seven** : **7×2=14**

(1) Define F–automorphism.

(2) Give an example of a finite field F such that $|F|$ is not a prime.

(3) Write down all the roots of the polynomial $x^4 - 2 \in Q[x]$.

(4) Give definition of a finite field extension. Also give an example of a finite field extension.

(5) Write down the minimal polynomial of $\cos\left(\frac{2\pi}{p}\right) + i.\sin\left(\frac{2\pi}{p}\right)$ over Q.

(6) Write down definitions of a prime field and give an example of a prime field.

(7) Write down definitions of separable polynomial, separable element and separable extension.

(8) State first fundamental theorem of R–homomorphism.

(9) Define exact sequence of R–modules and R–homomorphisms.

(10) Define cyclic field extension and give an example of finite cyclic extension.